Exercise 2.1.4.1

1. a. $\langle 2^n \rangle_{n=0}^{\infty} = 2^0 + 2^1 + 2^2 + 2^3 + \dots + 2^n$

1. b. $\langle x_n \rangle_{n=-1}^{\infty}$, where $\frac{3n+1}{|4n-1|}$

$$\langle x_n \rangle_{n=-1}^{\infty} = \frac{3(-1)+1}{|4(-1)-1|} + \frac{3(0)+1}{|4(0)-1|} + \frac{3(1)+1}{|4(1)-1|} + \frac{3(2)+1}{|4(2)-1|} + \dots$$

2. $x_0 = 3 \qquad x = \pi$

$$x_n = g\left(10^n \left(x - \sum_{k=0}^{n-1} 10^{-k} x_k\right)\right) \quad \text{for } n \geq 1$$

$$x_1 = g\left(10^1 \left(\pi - \sum_{k=0}^{0} 10^{-k} x_k\right)\right)$$

$$= g\left(10\left(\pi - [10^{-0} x_0]\right)\right)$$

$$= g(10\pi - 10x_0)$$

$$= g(10(\pi - 3))$$

$$= g(10(0.14\dots))$$

$$= g(1.4\dots)$$

$$= 1$$

(2)

$$x_2 = g(10^2(\pi - [10^{-0}(3) + 10^{-1}(1)]))$$
$$= g(100(\pi - 3 + \tfrac{1}{10}\cdot 1))$$
$$= g(100(\pi - 3 - 0.1))$$
$$= g(100(0.14\ldots - 0.1))$$
$$= g(100(0.04))$$
$$= g(4.\ldots)$$
$$= 4$$

$$x_3 = g(10^3(\pi - [10^{-0}(3) + 10^{-1}(1) + 10^{-2}(4)]))$$
$$= g(10^3(\pi - [3 + 0.1 + 0.04]))$$
$$= g(10^3(\pi - 3.14))$$
$$= g(10^3(0{,}00159\ldots))$$
$$= g(1{,}5\ldots)$$
$$= 1$$

and so on

$\langle x \rangle_n = 3, 1, 4, 1 \ldots$ looks like the sequence is perfect brownian motion hehe

③

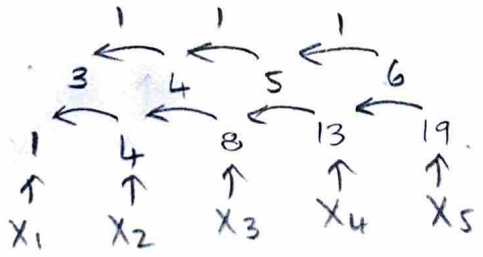
3a. $X_1 = 1, \quad X_{n+1} = X_n + 2$

1	3	5	7	9	
↑	↑	↑	↑	↑	
X_1	X_2	X_3	X_4	X_5	....

$\langle X \rangle_n$ clearly maps $\mathbb{N}$ to the odd numbers

$\langle X \rangle_n = 2n - 1$

b. $X_1 = 1 \qquad X_{n+1} = X_n + (n+1)$



The difference between each successive element is a sequence with a successive difference of 1. Hence, from some calculus knowledge, the function $\langle X \rangle_n$ is quadratic.

$$a1^2 + 1b + c = 1$$
$$a2^2 + 2b + c = 4$$
$$a3^2 + 3b + c = 8$$

This system of linear equations can be solved to get a, b, c such that $\langle X \rangle_n = an^2 + bn + c$